Nuclear symmetry energy effects on liquid-gas phase transition in hot asymmetric nuclear matter

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The liquid-gas phase transition in hot asymmetric nuclear matter is investigated within relativistic mean-field model using the density dependence of nuclear symmetry energy constrained from the measured neutron skin thickness of finite nuclei. We find symmetry energy has a significant influence on several features of liquid-gas phase transition. The boundary and area of the liquid-gas coexistence region, the maximal isospin asymmetry and the critical values of pressure and isospin asymmetry all of which systematically increase with increasing softness in the density dependence of symmetry energy. The critical temperature below which the liquid-gas mixed phase exists is found higher for a softer symmetry energy.

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The possible occurrence of liquid-gas phase (LGP) transition in intermediate energy heavy ion collisions using neutron-rich stable and future radioactive beams provides a rather unique tool to probe hot and dense phases of highly asymmetric nuclear matter. Collisions experiments [1, 2] with stable heavy nuclei at intermediate energy do indicate theoretically predicted [3] features of liquid-gas phase transition where the hot and compressed nucleus produced expands and fragments into several intermediate mass fragments (high-density liquid phase) and light particles and nucleons (low-density gas phase).

The early theoretical studies of the thermodynamic properties of liquid-gas phase transition [4–7] are mostly confined to symmetric nuclear matter that employed the quite well predicted [8–10] behavior of the symmetric nuclear matter equation of state (EOS). One of the major ingredients in studies of asymmetric nuclear matter require knowledge of the density dependence of symmetry energy $E_{\rm sym}(\rho)$ [11–13]. Unfortunately, the model predictions of $E_{\rm sym}(\rho)$ even for nuclear matter at zero temperature are extremely diverse [14]. Only at the nuclear saturation density $\rho_0 \approx 0.16$ fm^{-3} the value of $E(\rho_0, T=0) = 32 \pm 4$ MeV has been well constrained.

Recently some progress has been achieved by consistently constraining the symmetry energy of cold neutron-rich matter near normal matter density from analysis of isospin diffusion [11–13] and isoscaling [15] data in intermediate energy heavy ion collisions and from the study of neutron skin thickness of several nuclei [16, 17]. While knowledge of symmetry energy $E_{\rm sym}(\rho, T)$ at finite temperature in particular has received little attention [18–20] that is crucial for a proper understanding of the features of LGP transition in hot asymmetric nuclear matter. In fact, new qualitative features are expected when an asymmetric nuclear system with two conserved charges, baryon number and third component of isospin, undergoes a LGP change which has been suggested to be of second order [21]. Most previous studies of LGP transition [21–23] relied on model predictions of symmetry energy $E_{\rm sym}(\rho, T)$ with no or minimal contact with the available experimental data. Thus to understand better the features of LGP transition in hot asymmetric nuclear matter, it is imperative to employ the asymmetric nuclear EOS that has been constrained from analysis of skin thickness data of several nuclei [17] or from isospin diffusion/scaling data [12, 20]. Such an investigation is particularly useful as future experiments with radioactive ion beams with large neutron-proton asymmetries can be used to explore [24, 25] symmetry energy effects on liquid-gas phase transition.

In this letter, we study the effects of constrained symmetry energy [17] on the thermodynamic properties of LGP in hot neutron-rich nuclear matter within relativistic mean field (RMF) models [26]. For this purpose we use two accurately calibrated models: NL3 [27] and FSUGold [28], that was obtained by fitting the model parameters to certain ground state properties of finite nuclei. The interaction Lagrangian density in the nonlinear RMF model is given by [17, 25]

$$\begin{aligned}\mathcal{L} \;=\; & \overline{\psi}\left[g_s\phi - \left(g_v V_\mu + \frac{g_\rho}{2}\boldsymbol{\tau}\cdot\mathbf{b}_\mu + \frac{e}{2}\left(1+\tau_3\right)A_\mu\right)\gamma^\mu\right]\psi \\ & -\frac{\kappa}{3!}\left(g_s\phi\right)^3 - \frac{\lambda}{4!}\left(g_s\phi\right)^4 + \frac{\zeta}{4!}g_v^4\left(V_\mu V^\mu\right)^2 \\ & +\Lambda_v\left(g_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu\right)\left(g_v^2 V_\mu V^\mu\right) \qquad (1)\end{aligned}$$

which includes a isospin doublet nucleon field (ψ) interacting via exchange of isoscalar-scalar sigma (ϕ), isoscalar-vector omega (V^μ), isovector-vector rho ($\mathbf{b}^\mu$) meson fields and the photon (A^μ) field. The nonlinear sigma meson couplings (κ, λ) soften the symmetric nuclear matter EOS at around ρ_0, while its high density part is softened by the self-interactions (ζ) for the omega meson field.

For the original NL3 set with $\zeta = \Lambda_v = 0$, the saturation of symmetric nuclear matter occurs at a Fermi momentum of $k_F = 1.30$ fm^{-1} with a binding energy $B/A \approx 16.3$ MeV and an incompressibility of $K_0 = 271$ MeV. The original FSUGold [28], with two additional couplings $\zeta = 0.06$ and $\Lambda_v = 0.03$, with $K_0 = 230$ MeV produces a soft symmetric and asymmetric nuclear mat-

ter EOS. To study the effect of symmetric nuclear EOS (eg. incompressibility K_0) on the symmetry energy, we have extended [17] the original NL3 Lagrangian to include the isovector coupling Λ_v which is then varied along with g_ρ in both NL3 and FSUGold to generate various $E_{\rm sym}(\rho)$. All combinations of Λ_v and g_ρ are adjusted to a constant $E_{\rm sym}(\overline{\rho}, T=0) = 25.67$ (26.00) for the NL3 (FSUGold) at an average density $\overline{\rho}$ corresponding to $k_F = 1.15$ fm^{-1} where the binding energy of ^{208}Pb is reproduced. Thus the additional couplings provides an efficient way to change in a controlled manner the density dependence of nuclear symmetry energy without compromising the success of the model.

The model parameter (Λ_v, g_ρ) set is then varied to explore $E_{\rm sym}(\rho)$ effects on the liquid-gas phase transition in hot asymmetric nuclear matter. For the present study we use $\Lambda_v = 0.0-0.03$ since the resulting symmetry energies and their slopes and curvatures are in reasonable agreement with that extracted from neutron skin thickness of several nuclei as well as the isoscaling and isospin diffusion data [17]. It may be also noted that with increasing Λ_v the density dependence of symmetry energy becomes softer in both the NL3 and FSUGold models [17]. While at a finite Λ_v the symmetry energy $E_{\rm sym}(\rho, T=0)$ is found to be particularly stiff in FSUGold than in the NL3 parameter sets at densities $\rho \gtrsim 1.5\rho_0$.

At finite temperature and density the energy density $\mathcal{E}$ can be readily obtained from the thermodynamical potential Ω [21] as

$$\begin{aligned}\mathcal{E} &= \frac{2}{(2\pi)^3}\sum_{q=n,p}\int d^3k\, E^*(k)\left([n_q(k)]_+ + [n_q(k)]_-\right)\\ &+\frac{m_s^2\phi^2}{2} + \frac{\kappa}{3!}(g_s\phi)^3 + \frac{\lambda}{4!}(g_s\phi)^4 + \frac{m_v^2V_0^2}{2}\\ &+\frac{\zeta}{8}(g_vV_0)^4 + \frac{m_\rho^2 b_0^2}{2} + 3\Lambda_v (g_vV_0)^2(g_\rho b_0)^2, \end{aligned} \quad (2)$$

where $E^*(k) = \sqrt{k^2 + m^{*2}}$ is the effective energy. The distribution function for nucleon and antinucleon (referred to as $\pm$ sign)

$$[n_q(k)]_\pm = \frac{1}{\exp\left[\left(E^*(k) \mp \nu_q\right)/T\right] + 1} \quad (q=n,p), \quad (3)$$

where the effective chemical potential for neutron and proton is expressed as $\nu_q = \mu_q - g_vV_0 \pm g_\rho b_0/2$. The chemical potentials can be determined from the conserved baryon and isospin densities:

$$\rho = \frac{2}{(2\pi)^3}\int d^3k\,(G_p(k) + G_n(k)), \quad (4)$$

$$\rho_3 = \frac{2}{(2\pi)^3}\int d^3k\,(G_p(k) - G_n(k)), \quad (5)$$

where $G_q(k) = [n_q(k)]_+ - [n_q(k)]_-$.

As in the zero temperature case, several model studies [12, 19, 29, 30] have indicated that the EOS for hot neutron-rich nuclear matter can be expressed in the parabolic form:

$$E(\rho,T,\alpha) = E(\rho,T,\alpha=0) + E_{\rm sym}(\rho,T)\alpha^2 + \mathcal{O}(\alpha^4), \quad (6)$$

where the neutron-proton asymmetry is $\alpha = (\rho_n - \rho_p)/\rho$. The density and temperature dependence of symmetry energy can be estimated from $E_{\rm sym}(\rho,T) \approx E(\rho,T,\alpha=1) - E(\rho,T,\alpha=0)$. This implies that $E_{\rm sym}(\rho,T)$ is the energy required to convert all the protons in symmetric matter to neutrons. Figure 1 shows the density dependence of nuclear symmetry energy at temperatures $T = 0, 5, 10, 15$ MeV in the NL3 (left panel) and FSUGold (right panel) sets. For all choices of Λ_v the symmetry energy decreases with increasing temperature especially at small densities $\rho \lesssim \rho_0$ that is entirely due to the decrease in the kinetic energy contribution. For $\Lambda_v = 0.0$ (0.03) the density dependence of $E_{\rm sym}(\rho,T)$ at all temperatures exhibits a systematic trend of small (large) value at subsaturation densities and a large (small) value at supranormal densities resulting in an overall stiffer (softer) asymmetric nuclear matter EOS.

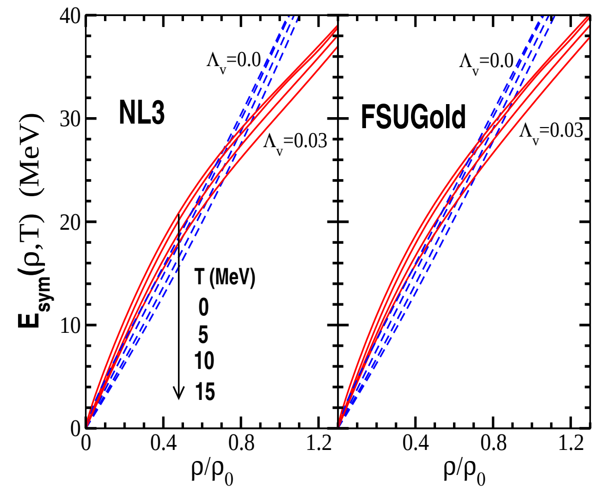

FIG. 1: Density dependence of nuclear symmetry energy at temperatures $T = 0, 5, 10, 15$ MeV in the NL3 (left panel) and FSUGold set (right panel) with couplings $\Lambda_v = 0.0$ and 0.03.

The above described models can now be used to study LGP in hot asymmetric nuclear matter. The system is stable against LGP separation if its free energy F is lower than the coexisting liquid (L) and gas (G) phases, i.e. $F(T,\rho) < (1-\lambda)F^L(T,\rho^L) + \lambda F^G(T,\rho^G)$ with $\rho = (1-\lambda)\rho^L + \lambda\rho^G$ where $0 < \lambda < 1$ and $\lambda = V^G/V$ being the fraction of the total volume occupied by the gas phase. The stability condition implies the inequalities [21]:

$$\rho\left(\frac{\partial P}{\partial \rho}\right)_{T,\alpha} > 0, \quad (7)$$

$$\left(\frac{\partial \mu_p}{\partial \alpha}\right)_{T,P} < 0 \quad \text{or} \quad \left(\frac{\partial \mu_n}{\partial \alpha}\right)_{T,P} > 0. \quad (8)$$

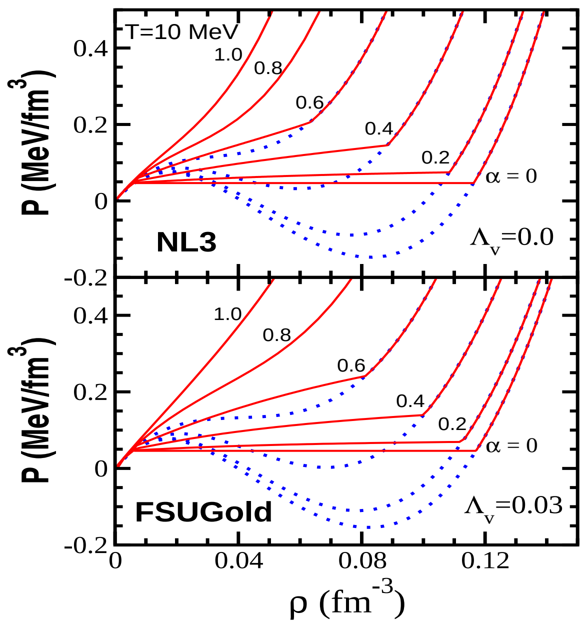

FIG. 2: Pressure as a function of density at temperature $T = 10$ MeV for various isospin asymmetry α in the original NL3 set [27] with $\Lambda_v = 0.0$ (top panel) and the original FSUGold set [28] with $\Lambda_v = 0.03$ (right panel). The dotted curves refer to unstable single phase while the solid curves refer to stable matter; see text for details.

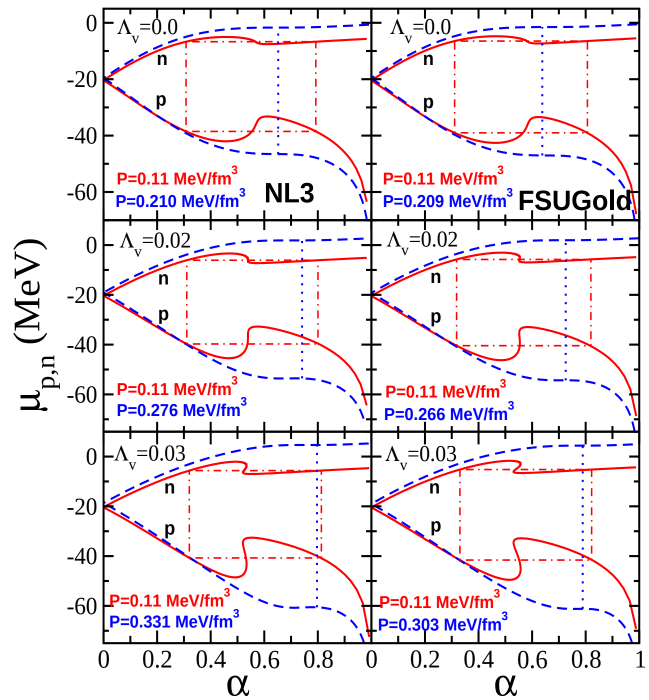

FIG. 3: Chemical potential isobars as a function of isospin asymmetry α at temperature $T = 10$ MeV for NL3 (left panel) and FSUGold (right panel) with different Λ_v couplings. The geometrical construction used to obtain the isospin asymmetries and chemical potentials in the two coexisting phases is also shown.

The first inequality indicates mechanical stability which means a system at positive isothermal compressibility remains stable at all densities. The second inequality stems from chemical instability which shows that energy is required to change the concentration in a stable system while maintaining temperature and pressure fixed. If one of these conditions get violated, a system with two phases is energetically favorable. The two phase coexistence is governed by the Gibbs's criteria for equal chemical potentials and pressures in the two phases with different densities but at the same temperature:

$$\mu_q^L(T, \rho^L) \;=\; \mu_q^G(T, \rho^G) \qquad (q = n, p), \tag{9}$$

$$P^L(T, \rho^L) \;=\; P^G(T, \rho^G). \tag{10}$$

Figure 2 shows the pressure as a function of nucleon density at a fixed temperature $T = 10$ MeV with different values of asymmetry α in the original NL3 and FSUGold sets. Below a critical value of asymmetry α, the pressure is seen (dotted curves) to decrease with density resulting in negative incompressibility and thereby a mechanically unstable system. The stable two-phase (liquid-gas) configuration at each density is obtained from Maxwell construction (solid lines). Analogues to intermediate energy heavy-ion collisions [1, 2] when the hot matter in the high density (liquid) phase expands it enters the coexistence LGP where the pressure decreases at a fixed $\alpha \neq 0$ for the two-component asymmetric matter. Whereas, for symmetric nuclear matter at $\alpha = 0$ the pressure remains constant at all densities. Finally the system leaves the coexistence region and vaporizes into the low density (gas) phase. Of particular interest here is the symmetry energy effects on the isotherms. It is clearly seen that in contrast to the original NL3 with $\Lambda_v = 0$, the softer $E_{\rm sym}(\rho)$ in the original FSUGold with $\Lambda_v = 0.03$ [17] enforces the onset of pure liquid phase to a higher density resulting in a wider coexistence region for each asymmetry α. Moreover, the critical pressure P_c above which the mixed liquid-gas phase vanishes is seen larger for this soft FSUGold set; a detailed discussion of which is presented below.

The details of chemical evolution for the LGP transition is depicted in Fig. 3 where the neutron and proton chemical potentials are shown as a function of isospin asymmetry α at a fixed $T = 10$ MeV and pressure $P = 0.11$ MeV/fm^3 for the NL3 (left panels) and FSUGold (right panels) at various Λ_v values. As usual, the bare nucleon mass has been subtracted from the chemical potentials. At fixed pressure and Λ_v, the solutions of the Gibbs conditions (9) and (10) for phase equilibrium form the edges of a rectangle and can be found by geometrical construction as shown in Fig. 3. At each Λ_v, the two different values of α defines the high density liquid phase boundary (with small $\alpha = \alpha_1(T, P)$) and the low density gas phase boundary (with large $\alpha = \alpha_2(T, P)$). From the figure it is evident that the symmetry energy

dependence of Λ_v in NL3 and FSUGold [17, 25] leads to different phase boundaries $\alpha_1(T,P)$ and $\alpha_2(T,P)$ and hence should predict different thermodynamic properties for the LGP transition.

As the pressure increases the system encounters a critical pressure P_c beyond which the matter is stable but below which the second inequality (8) gets violated and the system becomes chemically unstable. The critical pressure P_c is determined by the inflection point $(\partial\mu/\partial\alpha)_{T,P_c} = (\partial^2\mu/\partial\alpha^2)_{T,P_c} = 0$. The disappearance of chemical instability at P_c results in the neutron (proton) chemical potential to decrease (increase) with decreasing asymmetry α. Figure 3 also shows the chemical potential isobars at the critical pressure (dashed lines). The rectangle from Gibbs condition then collapses into a line vertical at $\alpha \equiv \alpha_c$. Correspondingly, (P_c, α_c) defines the critical point at a given temperature that refers to the upper boundary of instability with respect to pressure variation. Note at $T = 10$ MeV, the critical values (P_c, α_c) at $\Lambda_v = 0.0, 0.02, 0.03$ are respectively at (0.210, 0.652), (0.276, 0.741), (0.331, 0.797) for the NL3 set and at (0.209, 0.638), (0.266, 0.725), (0.303, 0.789) for the FSUGold set. Interestingly, we also find at a finite temperature the stiffness of symmetry energy has a significant influence on the phase-separation boundaries of LGP transition [20]. In general, a softer symmetry energy (larger Λ_v) gives systematically larger critical pressure and an enhanced asymmetry in the system. Moreover at a finite α, the relatively softer symmetry energy $E_{\rm sym}(\rho, T = 0)$ in the NL3 compared to FSUGold [17] translates to a larger critical pressures and asymmetry for the LGP transition.

All the pairs of solutions of Gibbs conditions, $\alpha_1(T,P)$ and $\alpha_2(T,P)$, form the phase-separation boundary or the binodal surface. In Fig. 4 we show the section of the binodal surface under isothermal compression of asymmetric nuclear matter at $T = 10$ MeV in the NL3 (top panel) and FSUGold (bottom panel). As expected the point of equal concentration (EC) corresponding to symmetric nuclear matter is independent of Λ_v. The critical point (CP) and EC divide the binodal section into two branches. One branch is the high-density (liquid) phase that is less asymmetric while the other branch corresponds to the more asymmetric low-density (gas) phase. Thus the matter on the left (right) of the binodal surface represents stable liquid (gas) phase. It is clearly seen here that the critical point (P_c, α_c) depends on the density dependence of the symmetry energy associated with different Λ_v values.

We also indicate on the binodal surface the maximal isospin asymmetry (MA), $\alpha_{\rm MA}$, of the system. Thus more neutron-rich matter on the right side of the surface when compressed/expanded at fixed α will never encounter a coexistence phase. Note here the maximal asymmetry is also quite sensitive to Λ_v i.e. on $E_{\rm sym}(\rho, T)$. Such effects found in the present study should have strong influence on the experimentally observed isospin distillation phenomena [31] where the gas phase is more neutron-

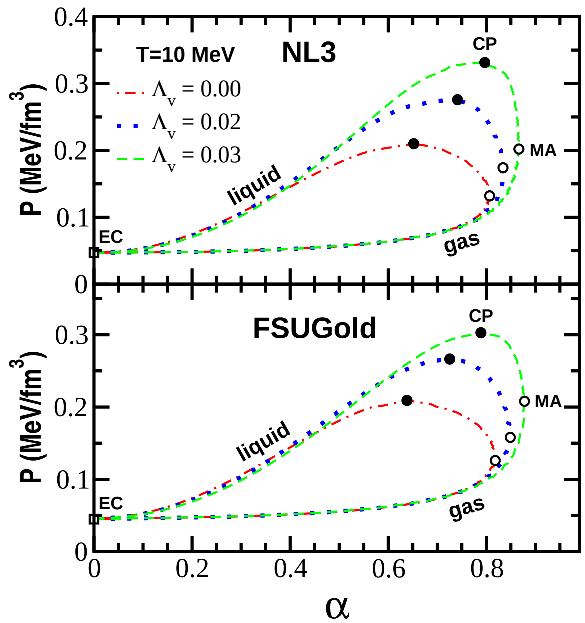

FIG. 4: The section of binodal surface at temperature $T = 10$ MeV in NL3 (top panel) and FSUGold (bottom panel) with different Λ_v couplings. The critical point (CP), the point of equal concentration (EC), and the maximal asymmetry (MA) are indicated.

rich (large n/p ratio) compared to the more asymmetric liquid phase. However for pressures $P \geq 0.10$ MeV/fm^3 the magnitude of isospin distillation is more sensitive to the symmetry energy used.

A new feature for LGP transition in asymmetric system, refereed to as retrograde condensation [21], arises when a nucleon gas prepared at an asymmetry $\alpha_c < \alpha < \alpha_{\rm MA}$ is compressed at fixed total α. The matter remains mechanically stable but chemically unstable. Thus a coexisting liquid phase emerges which finally vanishes when the system leaves the binodal surface as a pure gas. As the extent $\Delta\alpha = \alpha_{\rm MA} - \alpha_c$ is found to decrease for softer symmetry energy with higher Λ_v, the possibility of such unique-phase condensation phenomena also becomes minimal.

The present study clearly suggests that for liquid-gas phase transition in hot asymmetric nuclear matter, the critical values of pressure and isospin asymmetry, the maximal asymmetry and the area and shape of the binodal surface are quite sensitive to the density dependence of symmetry energy with a stiffer symmetry energy leads to consistently smaller values of these thermodynamic variables.

The existence of critical isospin asymmetry parameter α_c at a given temperature indicates that for $\alpha > \alpha_c$ the system will not change completely into the liquid phase. Conversely, this suggests that at a fixed α there exists a critical temperature T_c beyond which the system can only be in the gas phase at all pressures. In Fig. 5 we present T_c as a function of α in the FSUGold set for different

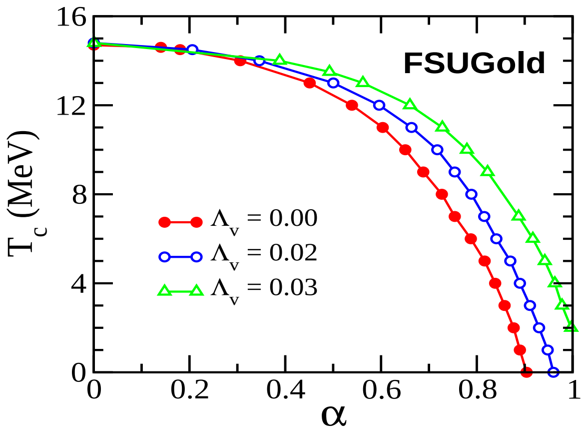

FIG. 5: The critical temperature T_c versus isospin asymmetry α for different Λ_v in the FSUGold set.

couplings Λ_v. For symmetric nuclear matter ($\alpha = 0$), the critical temperature for LGP transition in this model is $T_c = 14.7$ MeV. With increasing asymmetry $\alpha \gtrsim 0.6$, T_c decreases rapidly. A softer density dependence in symmetry energy (larger Λ_v) shows the coexisting liquid-gas phase can prevail for larger values of T_c. We find that for the soft symmetry energy ($\Lambda_v = 0.03$) even pure neutron matter ($\alpha = 1$) can exhibit LGP transition at $T \leq T_c = 2$ MeV. While the stiffest symmetry energy ($\Lambda_v = 0$) at $\alpha > 0.9$ predicts that the matter can only be in the pure gas phase at all temperatures.

In summary the effects of isospin symmetry interaction on the liquid-gas phase transition in hot neutron-rich nuclear matter is investigated. For this we have used the two accurately calibrated relativistic mean field models, the NL3 [27] and the FSUGold [28] wherein the density dependence of nuclear symmetry energy at zero temperature has been constrained within a limited range by neutron skin thickness data of several atomic nuclei. We find considerable sensitivity of the symmetry energy on the features of phase transition. Softer symmetry energies give progressively larger phase-separation boundaries with higher critical values for pressure and isospin asymmetry as well as maximal asymmetries. At a given asymmetry we find the critical temperature for the existence of the mixed liquid-gas phase increases with softer symmetry energy and predicts the possible occurrence of even an unstable pure neutron matter at finite temperatures.

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